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BRIEF COMMUNICATION

REFINEMENT OF THE MIXING-LENGTH MODEL FOR PREDICTION OF GAS–PARTICLE FLOW IN A PIPE

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1. INTRODUCTION

Gas–particle suspension flows are found in many industrial applications, such as cyclone separators and classifiers, pneumatic transport of powder, sand blasting, combustion of pulverized coal, as well as rocket exhausts containing ash or unburnt metal powders.

Despite considerable progress in analytical and experimental studies, the design of pneumatic transport systems largely relies on empirical correlations. This is due to the complex mechanisms involved in gas–particle, particle–particle and particle–pipe wall interactions. The gas–particle flow is characterized by couplings between two phases such as thermal coupling, momentum coupling and mass coupling.

An experimental study on gas–particle suspension flow in a pipe was conducted by Boothroyd (1966). More extensive experimental studies were undertaken by Lee (1982), Tsuji *et al.* (1984) and Cape & Nakamura (1973), who measured the pressure drop, velocity distribution and the friction factor.

As part of a numerical analysis of gas–particle flow, Crowe & Sharma (1978) developed a particle trajectory model. This model is based on treating the particles as sources of mass, momentum and energy to the gaseous phase. Crowe & Sharma (1978) also developed an implicit quasi one-dimensional numerical formulation for two-phase flow (CONVAS model), and Lee & Crowe (1982) extended the one-dimensional CONVAS model to two-dimensional flows (PSI-cell model).

Another approach in modeling gas–particle flow is to regard the conveying gas phase and particle phase as two interactive fluids which exchange momentum, energy and mass with each other. This is called the "two-fluid" equation model. Depending on the physical models about the interactive exchanges of momentum and energy, the pressure gradient and the constitutive relation between stress and strain in the particle phase, there have appeared in the literature several different derivations of the "two-fluid" equations (Drew 1971; Marble 1963).

Melville & Bray (1979) applied Owen's (1969) theory to analyze a gas–particle turbulent round jet, in which the bulk motion of the particles was treated as a hypothetically continuous fluid mixed with the conveying primary fluid.

Elghobashi & Abou-Arab (1983) developed two governing equations which describe the conservation of turbulent kinetic energy and the dissipation rate of that energy for the conveying fluid in a two-phase flow.

Choi & Chung (1983) modified the Melville & Bray (1979) model to apply it to a wall-bounded shear flow. Here, the relative velocity between the two phases was assumed to be negligible, and the mixing-length model was used. Such assumptions are valid only for small Stokes numbers, defined as the ratio of the aerodynamic response time to some characteristic time for the flow system. If the Stokes number is not small and if the mean velocity of the conveying fluid is slow, the particles are not able to respond quickly to changes in the gas flow, thus the relative velocity should not be neglected. In addition, it was assumed that the turbulent kinetic energy of the conveying fluid is generated by both the primary fluid and the fluctuating solid particles, but that it is dissipated by the primary fluid only.

Recently, it was shown analytically that the turbulent kinetic energy is dissipated by the relative motion between the two phases as well as by the primary fluid itself (Elghobashi & Abou-Arab 1983). Chung *et al.* (1985) developed an eddy-viscosity model based on an approximate balance equation for k derived from those of Elghobashi & Abou-Arab (1983). This model was successful in predicting the pressure drop of gas–particle flow in a venturi.

The objective of the present work is to analyze gas–particle flow in a pipe using this refined physical model. The present numerical results are compared with experimental results and previous predictions.

2. PHYSICAL MODELS FOR GAS–PARTICLE FLOWS

In the "two-fluid" equation model, mean continuity equations and volume-averaged momentum equations for the primary conveying fluid (gas phase) and secondary fluids (particle phase), neglecting the added mass effect of the accelerated fluid and Basset forces, are given in two-dimensional cylindrical coordinates (Boothroyd 1971) as follows:

primary fluid

$$\frac{\partial \bar{U}_f}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\bar{V}_f) = 0 \qquad [1]$$

$$\bar{\rho}_b \bar{U}_f \frac{\partial \bar{U}_f}{\partial x} + \bar{\rho}_b \bar{V}_f \frac{\partial \bar{U}_f}{\partial r} = -\frac{\partial P}{\partial x} + \bar{\rho}_b \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(v_{fl}\frac{\partial \bar{U}_f}{\partial r} - \overline{u_f' v_f'}\right)\right] + F_{px}; \qquad [2]$$

and

secondary fluid

$$\bar{U}_p \frac{\partial \bar{\rho}_p}{\partial x} + \bar{V}_p \frac{\partial \bar{\rho}_p}{\partial r} = -\frac{1}{r}\frac{\partial}{\partial r}(r\overline{\rho_p' v_p'}) - \frac{\bar{\rho}_p \bar{V}_p}{r} - \bar{\rho}_p \frac{\partial \bar{V}_p}{\partial r} - \bar{\rho}_p \frac{\partial \bar{U}_p}{\partial x}, \qquad [3]$$

$$\bar{\rho}_p \bar{U}_p \frac{\partial \bar{U}_p}{\partial x} + \bar{\rho}_p \bar{V}_p \frac{\partial \bar{U}_p}{\partial r} = -\alpha \frac{\partial p}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left[r\bar{\rho}_p v_{pl}\frac{\partial \bar{U}_p}{\partial r} - \overline{u_p' v_p'}\right)\right] \qquad [4]$$

$$- \overline{\rho_p' v_p'}\frac{\partial \bar{U}_p}{\partial r} - g(\rho_p - \alpha\rho_f) - F_{px},$$

$$\bar{\rho}_p \bar{U}_p \frac{\partial \bar{V}_p}{\partial x} + \bar{\rho}_p \bar{V}_p \frac{\partial \bar{V}_p}{\partial r} = -\frac{1}{r}\frac{\partial}{\partial r}(r\bar{V}_p\overline{\rho_p' v_p'}) - \frac{\partial}{\partial x}(\bar{U}_p\overline{\rho_p' v_p'}) \qquad [5]$$

$$- \frac{\partial}{\partial x}(\bar{\rho}_p\overline{u_p' v_p'}) - F_{pr}.$$

We assume that the local values of α are small enough (at most of the order of 10^{-3}) to cope with the assumption of a dilute suspension. Therefore the bulk density of the primary fluid $\bar{\rho}_b = \rho_f(1-\alpha)$ can be assumed to be constant, whereas the dispersed particle density ($\bar{\rho}_p$) is treated as a variable. It is assumed that the secondary fluid consists of particles of spherical shape and uniform size and that the interparticle collisions and interparticle interactions are negligible. The final terms F_{px}, in [2] and [4], and F_{pr}, in [5], represent the interaction force per unit volume between the two phases in the axial and radial directions, respectively. Then for the particle Reynolds number <700, F_{px} and F_{pr} can be written as follows (Boothroyd 1971):

$$F_{px} = (\bar{U}_p - \bar{U}_f)\frac{\bar{\rho}_p}{t^*} \qquad [6a]$$

and

$$F_{pr} = (\bar{V}_p - \bar{V}_f)\frac{\bar{\rho}_p}{t^*}, \qquad [6b]$$

where t^* is the Stokesian relaxation time; defined as the time required for a particle, released from rest into a following stream, to achieve 63% of the free stream velocity, and given by

$$t^* = \frac{\rho_s d_p^2}{18\,\mu f} \quad [7]$$

where f = correction factor ($= 1 + 0.15\,\mathrm{Re}_p^{0.687}$), d_p = particle diameter, ρ_s = density of material of the particle phase and μ = dynamic viscosity of the primary fluid. In [1]–[5] $\bar{U}$ and $\bar{V}$ denote the mean velocity components in the axial and radial directions, u' and v' the fluctuating turbulent velocity components and the subscripts f and p refer to the primary and secondary fluids, respectively. Symbols ν_{fl} and ν_{pl} stand for the laminar kinematic viscosity of the primary fluid and the virtual laminar kinematic viscosity of the secondary fluid, respectively.

For a simple gradient turbulence model at first-order closure level, the Boussinesq eddy-viscosity models are applied as in Choi & Chung (1983):

$$\overline{u_f' v_f'} = -\epsilon_f \frac{\partial \bar{U}_f}{\partial r}, \quad [8]$$

$$\overline{u_p' v_p'} = -\epsilon_p \frac{\partial \bar{U}_p}{\partial r}, \quad [9]$$

$$\overline{\rho_p' v_p'} = -\frac{\epsilon_p}{\sigma_\phi}\frac{\partial \bar{\rho}_p}{\partial r} \quad [10]$$

and

$$\overline{\rho_p' u_p'} = -\frac{\epsilon_p}{\sigma_\phi}\frac{\partial \bar{\rho}_p}{\partial x}, \quad [11]$$

where ϵ_f is the scalar eddy viscosity of the primary fluid, ϵ_p is that of the secondary fluid and ϵ_p/σ_ϕ is an eddy diffusivity of the secondary fluid. The Schmidt number, σ_ϕ, is chosen to be 0.7, as used by Melville & Bray (1979).

The turbulence models for the scalar eddy viscosities, ϵ_f and ϵ_p, were developed and tested successfully in a pipe flow by Choi & Chung (1983). But this model is only satisfied for small Stokes number. In the present study, a model used in Chung *et al.* (1985), based on approximate balance equations for k and derived from those of Elghobashi & Abou-Arab (1983), is used. Applying the simple mixing-length model to terms of the balance equations for k in a state of equilibrium, and by using the isotropic approximations, $k = 1.5\,u_f^2$ and $\epsilon = 0.08\,k^{3/2}/l_f$, the production term, P, and the dissipation term, η, of the turbulent kinetic energy are approximated as follows:

$$P = \bar{\rho}_f \epsilon_f \left(\frac{d\bar{U}_f}{dr}\right)^2 + 1.02 \bar{\rho}_f \epsilon_f^2 \frac{l_f}{u_f}\frac{d^2(1-\alpha)}{dr^2}\left(\frac{d\bar{U}_f}{dr}\right)^2 \quad [12]$$

and

$$\eta = 0.147\frac{\bar{\rho}_f u_f^3}{l_f} + C_{p\epsilon}\frac{\bar{\rho}_p}{t^*}\overline{(u_f - u_p)u_f}. \quad [13]$$

Since in the state of equilibrium $p = \eta$, the following balance equation may be obtained:

$$\bar{\rho}_f u_f l_f\left[1 - 1.02\, l_f^2 \frac{d^2(\bar{\rho}_p/\rho_s)}{dr^2}\right]\left(\frac{d\bar{U}_f}{dr}\right)^2 = 0.147\frac{\bar{\rho}_f u_f^3}{l_f} + C_{p\epsilon}\frac{\bar{\rho}_p}{t^*}\overline{(u_f - u_p)u_f}, \quad [14]$$

where u_f and u_p are the turbulent characteristic velocity scales for the primary and secondary fluids and l_f is the length scale for the primary fluid. The model constant $C_{p\epsilon}$ is of the order of unity.

Using the relations, $\epsilon_f = u_f l_f$ and $\epsilon_{f0} = l_f^2 |d\bar{U}_f/dr|$ in [14], the ratio of the scalar eddy viscosity in the particle-laden flow to that in the clean flow is found to be

$$\frac{\epsilon_f}{\epsilon_{f0}} = \left[\frac{6.8 - 6.8\, l_f^2 \dfrac{d^2}{dr^2}\left(\dfrac{\bar{\rho}_p}{\rho_s}\right)}{1 + C'_{pc} \dfrac{\bar{\rho}_p}{\rho_f} \dfrac{t_l}{t^*}\left(1 - \dfrac{\epsilon_p}{\epsilon_f}\right)} \right]^{1/2}, \qquad [15]$$

where C'_{pc} is a model constant to be determined and t_l is the Lagrangian integral time scale. For a case when $\bar{\rho}_p/\rho_s \ll 1$, the second term in the numerator in [15] may be neglected. The eddy viscosity of the clean fluid flow without suspension of solid particles, ϵ_{f0}, is modeled by the conventional mixing-length model (Choi & Chung 1983). The eddy viscosity ϵ_p and the virtual laminar kinematic viscosity ν_{pl} of the secondary fluid were estimated by Choi & Chung (1983) using the following models:

$$\frac{\epsilon_p}{\epsilon_f} = \frac{1}{1 + \alpha\left(\dfrac{t^*}{t_l}\right)^{\beta}} \qquad [16]$$

and

$$\frac{\nu_{pl}}{\epsilon_p} = \frac{\nu_{fl}}{\epsilon_f}. \qquad [17]$$

Here, $\alpha = 1$ and $\beta = 2$ are model constants and the turbulent time scale, t_l, is estimated by the relation $t_l = l_f^2/\epsilon_f$.

3. NUMERICAL PROCEDURE

A forward marching technique (Patankar & Spalding 1970) is used to solve the governing equation. Each equation is transformed by a stream function derived from the continuity equation of the primary fluid. In the Patankar & Spalding (1970) scheme, the pressure gradient is estimated from the information of the previous calculation step. Then, the momentum equation is solved for the next downstream step. In this case, due to the approximate estimate of the pressure gradient, the computed boundary is not coincident with the physical boundary. In order to minimize this discrepancy, they proposed the following formula to estimate the pressure gradient:

$$\frac{dp}{dx} = -\frac{F'}{A} - \frac{\dot{m}\bar{U}}{A^2}\frac{dA}{dx}, \qquad [18]$$

where F' is the retarding force ($F'/A = 2\tau_w/r$) per unit length exerted by the wall, τ_w and A are the wall shear stress and the cross-sectional area of the pipe, and $\dot{m}$ is the mass flow rate. The wall shear stress is generated by both the primary fluid and the secondary fluid (particles). The wall shear stress from the primary fluid is computed from the condition that the numerical value of $R_{2.5}$ should be matched at the joining point, as in the Patankar & Spalding (1970) scheme. The wall shear stress from the secondary fluid is obtained by using the mean velocity gradient of the secondary fluid at the wall. The initial velocity profiles of the primary and secondary fluids at the inlet of a pipe are assumed to be the same as when fully developed, as in Chung *et al.* (1985). Thus, the following relation is obtained for the case of $\bar{U}_p = \bar{U}_f$:

$$\frac{dp}{dx} = \frac{1}{r}\frac{d}{dr}\left[(\epsilon_{eff,f} + \epsilon_{eff,p})\, r \frac{d\bar{U}}{dr}\right], \qquad [19]$$

where ϵ_{eff} is the effective eddy viscosity, defined as the laminar kinematic viscosity plus the turbulent eddy viscosity. From [19] the fully developed velocity profile is obtained by substituting the turbulent model relations and integrating the equation with an assumed dp/dx estimated from Choi & Chung (1983). The initial particle density profile at the inlet of the pipe is assumed to be uniform across the section, and is taken from the given particle loading, $\bar{\rho}_p = Z\bar{\rho}_f$ (Z = loading ratio). The boundary conditions of $\bar{U}_f$, $\bar{V}_f$ and $\bar{V}_p$ at the solid wall are given by $\bar{U}_f = \bar{V}_f = \bar{V}_p = 0$. Assuming that the solid wall boundary is nonabsorbing and reflecting, the boundary condition of the density of the particle phase is taken to be $d\bar{\rho}_p/dr = 0$, as in Di Giacinto *et al.* (1982). The boundary condition of $\bar{U}_p$ at the wall is not certain and needs some elaboration, as follows. Since the particles are known to slip over the wall surface (Boothroyd 1971), $\bar{U}_p$ does not vanish at the wall. In addition, our basic assumption is that the overall pressure drop is due to the seconday fluid flow over the wall as well as the primary air flow. If the velocity gradient of the secondary fluid is assumed to be zero, our mixing-length model would negate any contribution of the secondary fluid to the overall pressure drop, which is in contradiction to our basic assumption. A reasonable boundary condition for U_p must be a compromise between the two logical extremes, $\bar{U}_p = 0$ and $d\bar{U}_p/dr = 0$. Careful inspection of the available experimental data of Lee (1982) and Tsuji *et al.* (1984) reveals that the gradient of the secondary fluid very near the wall is approximately proportional to that of the primary fluid, or $d\bar{U}_p/dr \propto d\bar{U}_f/dr$. A computer optimization finally gives the proportional constant to be about 0.8, which is almost a representative value of the above experimental data.

4. COMPUTATIONAL RESULTS AND DISCUSSIONS

In order to examine any improvement in predictions by the above refined mixing-length model, the present computations are compared with those by Choi & Chung (1983) on Boothroyd's (1966) experiments, in which the friction factors of air flows were measured in smooth pipes carrying spherical zinc powders. The pipe diameters were 0.0254, 0.0508 and 0.0762 m and the particle sizes were distributed over the range 0 to $\sim 40\,\mu m$, with an average size of $15\,\mu m$. Solid–gas loading ratios were between 0.3 and 10 and the Reynolds number, based on pipe diameter and the average velocity of the air, was fixed at 53,000 for all cases.

Figure 1 compares predicted friction factors with the experimental data, whose uncertainty is estimated to be within 10%. The dashed curves are the predictions by Choi & Chung (1983) and

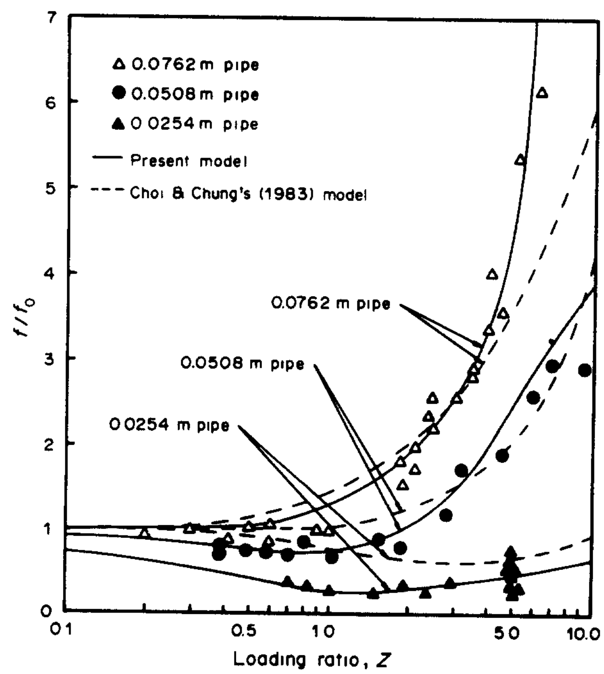

Figure 1. Comparison of predicted friction factors with the experimental data of Boothroyd (1966).

the solid curves are the present results. As can be seen, the former do not agree with the experimental data for larger relative particle size, whereas the present predictions show excellent agreement with them. This may be attributed to the fact that Choi & Chung's (1983) eddy-viscosity model is satisfied only when the Stokesian relaxation time is much smaller than the Lagrangian integral time. It can be shown also that for a given Reynolds number and solid–gas loading ratio, the wall friction factor increases as the relative particle size decreases. Figure 2 shows the solid friction factor, defined by

$$f_p = \frac{2D\Delta P_s}{\bar{\rho}_p \bar{U}_p^2 L}, \qquad [20]$$

where ΔP_s and D are the frictional pressure drop due to the particles and the pipe diameter, respectively and $\bar{U}_p$ is the mean velocity of the particles. The dashed curve is Yang's (1978) correlation. Yang assumed that pneumatic conveying behaves similarly to moving beds if the relative velocities are used in place of the fluid velocities and developed a correlation formula for the solid friction factor as follows:

$$f_p \frac{(1-\alpha)^3}{\alpha} = 0.0126\left(\alpha \frac{\mathrm{Re}_t}{\mathrm{Re}_p}\right)^{-0.979}, \qquad [21]$$

where $(\mathrm{Re})_t$ is the Reynolds number defined as $(\rho_f U_t d_p)/\mu$ and U_t is the terminal velocity of a single particle. The experimental data used in the formulation of [21] were taken from experiments with much larger relative velocities between the two phases compared with those in the present analysis

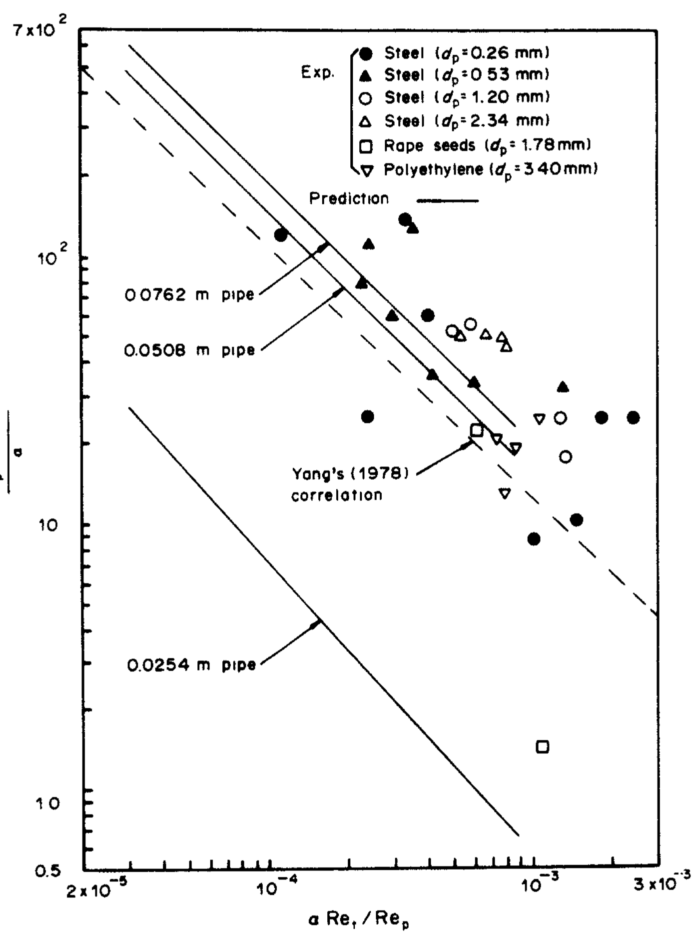

Figure 2. Correlations between the solid friction factor and a Reynolds number ratio, $\alpha\,\mathrm{Re}_t/\mathrm{Re}_p$.

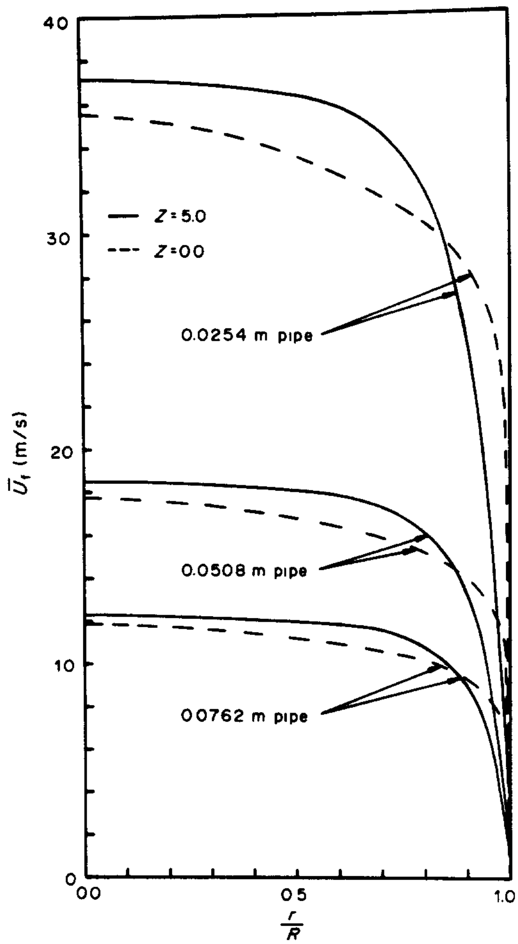

Figure 3. Mean velocity profiles of the primary fluid for various pipe sizes and gas–solid loading ratios.

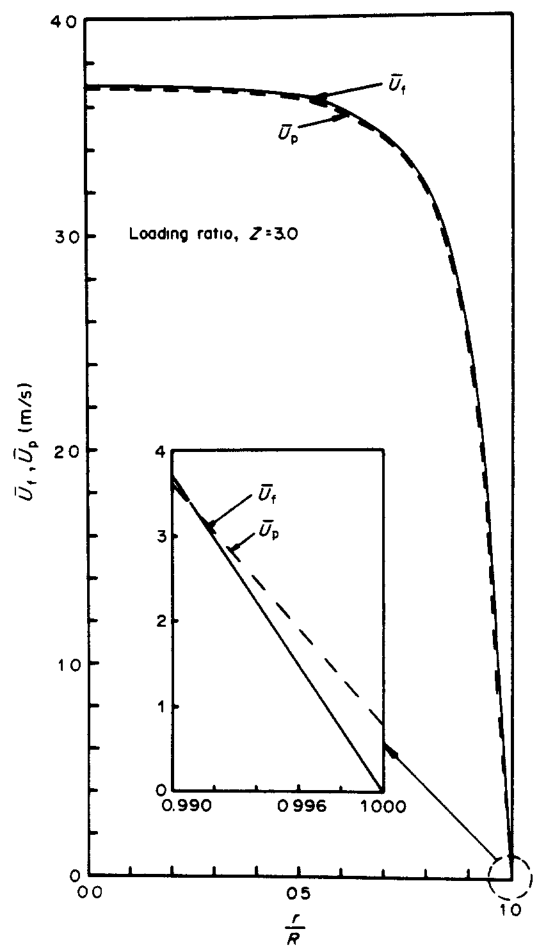

Figure 4. Predicted mean velocity profiles of the primary and secondary fluids in a pipe.

(5 to ~10 m/s). The solid curves were obtained by our computation, which has almost the same slope as the correlation [23] but its magnitude is strongly dependent on the relative particle size.

The mean velocity profiles of the primary fluid in 0.0254, 0.0508 and 0.0762 m pipes are shown in figure 3 with the solid–gas loading ratio as a parameter. When this ratio increases, the velocity profiles become more rounded in the core region. The variation of the velocity profile is more significant for larger relative particle size. Such an increase in the maximum velocity at the pipe center for higher loading ratios has also been observed in the experiments of Gill *et al.* (1964).

The computed mean velocity profiles of the primary and secondary fluids are compared in figure 4. The relative velocity between the two phases becomes zero at a distance very close to the wall and then particle phase precedes the primary fluid near the wall, which is in agreement with experimental observations by Lee (1983).

5. CONCLUSIONS

A "two-fluid" model using the refined mixing-length theory has been applied to investigate turbulent dilute gas–particle flow in a pipe. Eddy-viscosity models for gas–particle flow in a pipe and for the primary and the secondary fluids have been derived from an approximate balance equation for the turbulent kinetic energy in a state of local equilibrium. Turbulent kinetic energy is assumed to be dissipated by the fluctuations of the particle phase as well as by the primary conveying fluid. The boundary condition of the particle phase at the solid wall was chosen such that it permits a mean relative velocity at the wall, as observed experimentally. The Stokesian drag force terms, due to nonzero relative velocity between the two phases, were included in the governing equations for both phases. The present computational model predicts the characterizations of gas–particle flow much better than the previous model over a broader range of Stokes number, loading ratio and relative particle size.

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